Do Government Subsidies Increase the Private Supply of Public Goods?

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Current version: preprint, 1995

Abstract. We study three different models in which public goods are supplied by private contributions. In one of these models, tax-financed government subsidies to private contributions will definitely increase the equilibrium supply of public goods. In the other two models, government subsidies are neutralized by offsetting changes in private contributions. We explain why it is that these models lead to opposite conclusions and we argue on the basis of our first model that a government that wants to use taxes and subsidies to increase total provision of public goods will be able to do so. Indeed, our model yields a surprisingly decisive comparative statics result. If public goods and private goods are both normal goods, then increases in the subsidy rate necessarily increase the equilibrium supply of public goods.

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Peter Warr (1983) discovered the remarkable fact that in a Nash-equilibrium model of voluntary public goods supply, small income redistributions among contributors to a public good are "neutralized" by changes in equilibrium private donations. Private consumption by individuals and the supply of public goods remain exactly the same as before redistribution. Warr (1982) also observed that small government contributions to a public good, paid for by arbitrary lump sum taxes on contributors, would be offset "dollar-for-dollar" by reductions in private contributions. The comparative statics of non-infinitesimal redistributions and of economies where some contributors may make zero contributions were examined by Bergstrom, Blume, and Varian (1986). They showed that redistribution is not, in general, neutral if the amount of income distributed away from any consumer exceeds his voluntary contribution to the public good.

Russell Roberts (1987, 1991) suggested that if public goods are paid for by distortionary taxes, then efficiency would be improved by a "mixed system" in which the public good is supplied by private contributions that are (perhaps very heavily) subsidized by the government. While this idea seems appealing, it must somehow be reconciled with apparently contradictory results of Douglas Bernheim (1986), and of James Andreoni (1988), each of whom extended Warr's result to show that seemingly distortionary taxes and subsidies may be "neutralized" by changes in private donations.

This paper examines the neutrality results found by Andreoni and by Bernheim, and presents a new model in which increases in subsidy rates will *necessarily* increase the equilibrium supply of public goods. We show that in all of these models neutralization of tax-subsidy schemes is limited to "small" changes in tax obligations that do not exceed anyone's voluntary contributions in the original equilibrium. Moreover, we argue that it is possible for governments to design "distortionary" tax-subsidy policies which can predictably increase the equilibrium total amount of contributions to a public good.

1. Game 1–A Model Where Government Subsidies are not Neutral

Here and in subsequent sections, we follow the useful precedent set by Bernheim (1986) and Nett and Peters (1990) of describing the economy as a multi-stage game, with distinct stages in which government sets policy parameters, consumers choose actions, and the government collects taxes and provides public goods.

The economy has n consumers, one private good and one pure public good. Preferences of consumer i are represented by a utility function, $U_i(g, c_i)$, where g is the amount of public

This work was begun while the authors were both attending the Warwick Summer Research Workshop in Economics in July 1991. The authors are grateful to Clive Fraser of the University of Warwick for helpful comments.

good provided and c_i is i's private consumption. Consumer i has an initial endowment of m_i units of private good. The public good is produced from private goods at a cost of one unit of private good per unit of public good. The public good is supplied by voluntary private contributions, which may be subsidized by the government. Let g_i be the amount of public good donated by consumer i. Total supply of public goods is then $g = \sum_1^n g_i$. Let $g_{\sim i} = g - g_i$ denote contributions by consumers other than i. The government subsidizes voluntary contributions at the rate β, where $0 < \beta < 1$. Thus, if consumer i contributes g_i units of public good, he will receive a payment of βg_i from the government. The cost of the government subsidies is paid with a system of taxes such that consumer i is taxed for a fixed share s_i of total government outlays, where $s_i \geq 0$ for all i and $\sum_1^n s_i = 1$. The government's total expenditure on subsidies is βg, and consumer i's tax bill is $s_i \beta g$. The budget constraint of consumer i is

$$c_i + (1-\beta)g_i = m_i - s_i\beta g. \tag{1}$$

with the additional constraint that $g_i \geq 0$.

The game has three stages. In stage 1, the government chooses the subsidy rate β, and tax shares $s_1, \ldots, s_n$. In stage 2, consumers simultaneously choose their contributions g_i so as to maximize their utilities subject to the budget constraints in Equation 1. In stage 3, the government observes the vector $g_1, \ldots, g_n$ and collects taxes $s_i\beta g$ from each i and pays a subsidy, βg_i, to each i. [1]

We study Nash equilibrium for consumers in stage 2. In equilibrium, each consumer's choice of g_i is his best response, given the total contributions $g_{\sim i}$ of others. Each i chooses g_i to maximize $U_i(g_{\sim i} + g_i, c_i)$ subject to Equation 1 and subject to $g_i \geq 0$. Notice that if consumer i believes that his choice of g_i does not change the contributions of others, then he must believe that his choice of g_i will determine the variable g and will also determine his tax bill $s_i\beta g$ and his subsidy payment βg_i. Equation 1 can be rearranged as follows:

$$c_i + (1 - \beta(1 - s_i))g = m_i + (1-\beta)g_{\sim i}. \tag{2}$$

In equilibrium, each consumer i's choice of g_i is equivalent to choosing g to maximize $U_i(g, c_i)$ subject to Equation 2 and subject to the constraint that $g \geq g_{\sim i}$.

Let us define the "demand functions," $G_i(p, y)$ and $C_i(p, y)$, so that $G_i(p, y)$ and $C_i(p, y)$ are the choices of g and c_i that maximize $U_i(g, c_i)$ subject to the budget constraint $c_i + pg = y$. Following the standard definition of normal goods in consumer theory, we say that *private and public goods are both normal goods* for consumer i if $G_i(p, y)$ and $C_i(p, y)$ are both strictly increasing functions of y.[2]

[1] For the purposes of this model, the Nash outcome would be the same if stage 3 were collapsed into stage 2, with the government and the consumers playing simultaneously. But it may be easier to think of there being a third stage.

[2] Implicit in this construction is the assumption that both functions are well-defined and single-valued. This amounts to assuming that preferences are continuous and strictly convex.

When public and private goods are both normal goods, we have a remarkably strong and decisive result. Not only does a unique equilibrium exist for any given subsidy rate, but the amount of public goods supplied is an unambiguously increasing function of the subsidy rate.

Theorem 1. *(Existence and Uniqueness.) If preferences are continuous and strictly convex and if public goods and private goods are normal goods, then for any* β *such that* $0 \leq \beta < 1$ *and for any* $s_1, \ldots, s_n$*, such that* $0 \leq s_i < 1$ *for all* i *and such that* $\sum_1^n s_i = 1$*, there exists exactly one Nash equilibrium* $g_1, \ldots, g_n$*.*

The proof of Theorem 1 is provided in the Appendix. While other authors have shown existence and uniqueness with similar models, our proof is new and, we think, of some interest.

Theorem 2. *(Monotonicity in Subsidies.) If the assumptions of Theorems 1 hold and if in addition, all consumers* i *have "smooth" indifference curves, then the larger is the subsidy rate* β*, the greater will be the amount of public goods supplied.*

Readers who just want to see how the story comes out may choose to skip to the next section, while those with strong interests in public goods provision may find it useful to follow the proof of Theorem 2. Many writers have avoided dealing with comparative statics with corner solutions on grounds of "tractability." This proof, we hope, will show that sharp and elegant comparative statics results can be found without the assumption that equilibrium is interior.

In order to prove Theorem 2, we establish a lemma that is a rather interesting general proposition in consumer theory. Stated informally this result is as follows: If there are two goods and both are normal, then if the price of one good falls and income changes in any way whatsoever, it must be that either (a) demand increases for the good whose price falls or (b) demand decreases for the good whose price stays constant (or possibly both).[3]

Lemma 1. *Let all consumers* i *have strictly convex preferences and "smooth" indifference curves at* (g, c_i)*.* [4] *If both goods are normal goods and* $p' < p$*, then for any* y *and* y'*, either* $G_i(p', y') > G_i(p, y)$ *or* $C_i(p', y') < C_i(p, y)$*.*

Proof of Lemma 1.

Let $p' < p$, and let $g' = G_i(p', y')$, $g = G_i(p, y)$, $c' = C_i(p', y')$, and $c = C_i(p, y)$. By the principal of revealed preference it must be that $G_i(p', p'g + c_i) > G_i(p, y)$ and $C_i(p', p'g + c_i) < C_i(p, y)$. There are two possibilities. Either $p'_i g + c_i \leq y'$, or $p'_i g + c_i > y'$. In the former case, it follows from normality of g that $G_i(p', y') \geq G_i(p', p'_i g + c_i) > G_i(p, y)$.

[3] The lemma holds when both goods are private goods, as well as when one is public.

[4] Indifference curves are smooth at a point if there is no more than one tangent line at that point. This rules out "kinked" indifference curve. Without smoothness, a weaker version of Lemma 1 would obtain, where the conclusion holds with weak (but not strict) inequalities.

In the latter case it follows from normality of c_i that $C_i(p', y') < C_i(p', p'_i g + c_i) < C_i(p, y)$. ∎

Proof of Theorem 2.

Consider the Nash equilibria that correspond to two different subsidy rates, β and β', where $\beta' > \beta$. For each i, define $p_i = 1 - \beta(1 - s_i)$ and $p'_i = 1 - \beta'(1 - s_i)$. Then $p'_i < p_i$. Let $y_i = m_i + (1 - \beta)g_{\sim i}$ and let $y'_i = m_i + (1 - \beta')g'_{\sim i}$. Let S be the set of consumers for whom $g_i > 0$.

Suppose that $g' \leq g$. If for some consumer $i \in S$, $G_i(p', y'_i) > G_i(p, y)$, then $g' \geq G_i(p', y') > G_i(p, y) = g$. Therefore if $g' \leq g$, it must be that for all $i \in S$, $G_i(p', y'_i) \leq G_i(p, y)$. It follows from Lemma 1, that if $g' \leq g$, then $C_i(p', y'_i) < C_i(p, y)$ for all $i \in S$. Let $c'_i = C_i(p', y'_i)$ and $c_i = C_i(p, y)$. According to the budget equation (1), for all $i \in S$, $c_i = m_i - (1-\beta)g_i - s_i\beta g$ and $c'_i = m_i - (1-\beta')g_i - s_i\beta g'$. If $c'_i < c_i$ for all $i \in S$, it must be that $\sum_S c'_i = \sum_S (m_i - (1-\beta')g'_i - s_i\beta g') < \sum_S c_i = \sum_S (m_i - (1-\beta)g_i - s_i\beta g)$ and hence $(1-\beta)\sum_S g_i + \sum_S s_i\beta g < (1-\beta')\sum_S g'_i + \sum_S s_i\beta' g'$. But $\sum_{i\in G} g_i = g$ and $\sum_{i\in G} g'_i \leq g'$. Therefore $(1-\beta+\sum_S s_i)g < (1-\beta'+\sum_S s_i)g'$. Since $0 < (1-\beta'+\sum_S s_i) < (1-\beta+\sum_S s_i)$, it must be that $g' > g$. But this contradicts our initial assumption that $g' \leq g$. It follows that $g' > g$. ∎

2. Game 2–A Game with Neutral Subsidies.

In this section we consider an alternative model, due to Andreoni (1988), in which taxes and small government subsidies to public goods are neutralized in equilibrium. Although, as we will see, the applicability of neutrality is limited to relatively small changes in tax rates, it is interesting to see just how it happens that fiscal policies that seem capable of altering the allocation of resources are neutralized in equilibrium.

Let preferences and technology be as in Game 1. As in Game 1, the government chooses a subsidy rate β at which it will subsidize private contributions to public goods. But in this game, the government's tax policy is different. In Stage 1, the government chooses a "head tax" τ_i to be assessed against each i and a subsidy rate β which will be paid on private contributions to public goods. Thus a consumer who contributes g_i units of public goods will receive a subsidy of βg_i and will have a net tax obligation of $\tau_i - \beta g_i$. The government uses its net revenue, which is $\sum_i(\tau_i - \beta g_i)$, to supply additional units of the public good.

In Stage 2 of the game, the consumer's budget equation is $c_i + g_i = m_i - \tau_i + \beta g_i$ or equivalently,

$$c_i + (1 - \beta)g_i = m_i - \tau_i. \tag{3}$$

The supply of public goods is the sum of individual contributions plus the government's contribution. This gives us $g = g_i + g_{\sim i} + \sum_j \tau_j - \beta\sum_j g_j$. This equation can be rewritten as $(1-\beta)g_i = g - \sum_i \tau_i - (1-\beta)g_{\sim i}$. Substituting this expression into the budget constraint in Equation 3, we can write the budget equation in "standard form", much as we did for

Game 1 in Equation 2. This budget is

$$c_i + g = m_i + \sum_{j \neq i} ((1-\beta)g_j - \tau_j). \tag{4}$$

For Game 2, as for Game 1, Nash equilibrium exists and is unique under quite weak assumptions. The following is proved in the appendix.

Theorem 3. *(Existence and Uniqueness.) If preferences are continuous and strictly convex and if public goods and private goods are normal goods, then for any subsidy rate β such that $0 \leq \beta < 1$ and head taxes $\tau_1, \ldots, \tau_n$, such that $\tau_i < m_i$ for all i, there exists exactly one Nash equilibrium $g_1, \ldots, g_n$.*[5]

For this game it turns out that sufficiently "small" change in the subsidy rate β and the head taxes τ_i will be neutralized by offsetting private actions. This happens because each individual is able to adjust his private contributions to the tax-subsidy schedule in such a way that *no matter what the other players do*, his own private consumption will be the same as before the change.

Let c_i^* and g_i^* be consumer i's equilibrium levels of private consumption and public contribution if taxes and subsidies are both zero. Then $c_i^* + g_i^* = m_i$ and (c_i^*, g^*) maximizes $U_i(c_i, g)$ subject to $c_i + g \leq m_i + g_{\sim i}$. From Equation 2 it follows that with subsidy rate β and tax rate τ_i, consumer i can maintain a consumption of c_i^* by setting

$$g_i = (g_i^* - \tau_i)/(1-\beta). \tag{5}$$

If all consumers $j \neq i$ maintain consumption levels c_j^* by choosing $g_j = (g_j^* - \tau_i)/(1-\beta)$, then $\sum_{j \neq i}((1-\beta)g_j - \tau_j) = g_{\sim i}^*$, so that the budget equation (4) simplifies to $c_i + g = m_i + g_{\sim i}^*$. This is precisely the budget equation faced by i when there are no taxes and subsidies. Therefore i will choose to maintain the initial public goods supply g^*, by setting his own gift equal to $g_i = (g_i^* - \tau_i)/(1-\beta)$ and keeping $c_i = c_i^*$. Therefore, despite the taxes and subsidies, there is a Nash equilibrium in which private consumptions and public goods are the same as in the no-tax, no-subsidy equilibrium. We have proved the following.

Theorem 4. *(Neutrality) In Game 2, let $g_1^*, \ldots, g_n^*$, be the Nash equilibrium contributions if taxes and subsidies are zero. If the government introduces taxes and subsidies such that $\tau_i \leq g_i^*$ for all i, then in the new equilibrium with taxes and subsidies, each consumer will have the same private consumption as in the original equilibrium and the total amount of public goods will also be unchanged.*

Given the uniqueness result of Theorem 3, it follows from Theorem 4 that when taxes and subsidies can be offset by changes in private contributions, then the *only* Nash equilibrium is one in which the original consumptions are unchanged.

[5] The equilibrium could possibly be one in which the government's contribution to the supply of public goods is negative. As a formal matter, the model still makes sense if we allow the government to run a deficit and "pay for" its subsidy program by selling off private donations to the public good.

From Theorem 4 and its proof we can understand the exact limitations of the scope of neutrality to "small" changes in taxes and subsidies. Equation 5 informs us that consumer i can maintain a constant consumption after the introduction of taxes and subsidies if and only if $g_i^* \geq \tau_i$. This is the case if and only if the head tax that is introduced to pay for the subsidies does not exceed any consumer's initial voluntary contribution.

The fact that neutrality fails for large changes in taxes and subsidies should not be regarded as a mere technical curiosity. To appreciate this point, notice that in a large economy, if a pure public good is supplied by voluntary contributions, the equilibrium supply of public goods will be far short of the Pareto optimal supply. The government could surely choose to collect taxes and to supply an amount of public goods that is close to Pareto efficient. Of course this would change the real allocation of resources, and so government's actions must not be neutralized. The neutrality that holds for small changes fails as soon as tax collections from some individuals come to exceed the amount of public goods that they would contribute voluntarily.

3. Game 3–Bernheim's Game with Neutral "Distortionary Taxes"

Bernheim (1986) presents a different model in which seemingly distortionary government policies are neutralized in equilibrium. According to Bernheim, if "individuals care about the magnitude of their own contributions only insofar as these contributions affect the aggregate level of expenditures; and all individuals make positive contributions", then (among other things) "any policy consisting of apparently 'distortionary' transfers and distortionary public finance of the privately provided public good, has no effect on resource allocations."

In Bernheim's model, distortionary transfers take the form of taxes on labor income, which distort the labor-leisure choice. As Bernheim acknowledges, his result depends crucially on the way that decisions are timed. We will discuss a simplified variant of Bernheim's model which preserves the features essential to his argument, but in its simplicity is more transparent than the original.

The economy has n consumers and three commodities–an ordinary consumption good, leisure, and a pure public good. Preferences of consumer i are represented by a utility function of the form $U_i(g, c_i, \ell_i)$, where g is the total amount of public good provided, c_i is i's consumption of the private good and ℓ_i is i's leisure.

Person i's income depends on how much leisure he chooses, according to a function $m_i(\ell_i)$. From consumer i, the government collects an amount of taxes which depends on the leisure choices, $\ell = (\ell_1, \ldots, \ell_n)$, according to a function, $t_i(\ell)$. Each unit of public goods costs one unit of private goods and the government spends all of its tax revenue on the public good. Therefore, the amount of public good provided by the government is $g_0 = \sum_1^n t_i(\ell)$. Individuals may also make voluntary contributions toward the supply of public good, where g_i is the contribution of consumer i. Total supply of public goods is then $g = \sum_0^n g_i$. Let $g_{\sim i} = g - g_i$ denote total contributions by the government and by consumers other than i.

Following Bernheim, we model the economy as a game with four stages. In the first stage, the government chooses tax functions, $t_i(\ell)$. In the second stage, consumers choose their amounts of leisure. In the third stage, consumers choose their voluntary contributions to the supply of public goods. In the fourth stage the government collects the voluntary contributions and taxes according to the functions t_i, and supplies an amount of public goods equal to its total revenue.

Since the vector ℓ of leisure choices is determined in Stage 2, it must be that when Stage 3 begins, each consumer's tax bill, after-tax income, and leisure is already set. Therefore in Stage 3, the only choice left to a consumer is the division of his after-tax income between private consumption and contributions to the public good. This means that Stage 3 of this game is equivalent to Stage 2 of the game studied in the first section of this paper where the subsidy rate β is zero and where $m_i = m_i(\ell_i) - t_i(\ell)$, for each i. Therefore Theorems 1 and 2 establish existence and uniqueness of Stage 3 equilibrium conditional on the outcome, ℓ, of Stage 2.

Bernheim shows that subject to certain qualifications, if the government imposes a non-neutral tax on labor income and uses it to finance government supply of public goods, then in equilibrium, the government's activities will be undone by offsetting changes in consumer actions. Stated informally, the reasoning is this. Suppose that after the government introduces a tax, every consumer decides to take exactly as much leisure as he did when there was no tax and also decides to reduce his voluntary contribution by the amount of the tax. Then each consumer will be consuming the same amount of private goods as he was before the tax was imposed. The amount of public goods supplied is the sum of private contributions and tax revenue, so it too will be unchanged. If all other consumers choose not to change their leisure and to reduce their donations by the amount of the tax, then consumer i's budget constraint allows him to pursue the same strategy. Furthermore, the set of combinations of private consumption and public goods supply that are possible for him is contained in the combinations available to him when there is no tax. Therefore his best choice will be the same combination of private consumption and public goods supply that he chose when there were no taxes.

More formally, we have the following, which is proved in the Appendix.

Theorem 5. *In Game 3, consider a Nash equilibrium in which the government collects no taxes and pays no subsidies, and let* $\ell^* = \ell_1^*, \ldots, \ell_n^*$ *and* $g_1^*, \ldots, g_n^*$*, be the equilibrium vectors of work efforts and donations to the public good. If the government introduces taxes according to a schedule* $t_i(\ell)$*, such that* $t_i(\ell^*) \leq g_i^*$ *for all* i*, then there exists a subgame-perfect Nash equilibrium for Game 3, in which each consumer enjoys the same work effort, the same consumption, and the same amount of public goods as in the no-tax equilibrium.*

Theorem 5 would not be true without the assumption that $t_i(\ell^*) \leq g_i^*$ for all i. This requirement means that the neutrality result only applies to the introduction of taxes in such a way that nobody is assessed a tax greater than the amount of money he had been contributing voluntarily to the supply of public goods.

4. So What is the Answer?

In Game 1, the government is able to change the real allocation of resources in a predictable way. In Games 2 and 3, seemingly distortionary government policies have no real effect on equilibrium. What, then, should we conclude about whether government policies are neutral?

In all three of the games that we modeled, if the government imposes a small lump sum tax on a contributor and spends the proceeds on the public good, there will be no real effect. When the government takes money from a person's pocket and gives it to the public good, she is not "fooled". She will simply reduce her voluntary contributions by the amount of the lump sum tax. If people are prescient enough to see through the government veil of lump sum taxes, then we might wonder why shouldn't they also be able to see that "distortionary taxes" are also a veil? After all, the government policy does not create any new wealth. If people can obtain their initial no-tax demands by offsetting changes in their contributions, won't they try to do so, even with a "distortionary" tax? The prediction of a neutral subsidy has some appeal to our economist's sense that rational agents who are able to see through the veil of government policy, will also be able to discard this veil.

Our first response to this question is to observe that even in those models where neutrality is found, the scope of neutrality is very limited. Individuals are able to unravel the effect of a change in their tax burden only if the tax assessed against them does not exceed the amount that they would contribute voluntarily to the public good in the absence of taxes. The neutrality results in our Games 2 and 3, like those found by Warr, Andreoni, and Bernheim all fail if any individual is taxed in excess of the amount he would contribute voluntarily.

What remains to be understood is the difference that leads to "local" neutrality in Games 2 and 3, but not in Game 1. We will examine the difference between Games 1 and 2. Similar considerations apply to the comparison of Games 1 and 3. In Game 1 the government commits to making no *direct* contributions of public goods.[6] The government is assumed to balance the budget by adjusting taxes on individuals. By contrast, in Game 2 the government commits to no change in taxes, and balances the budget by adjusting its contribution to the public good. In Game 2, each consumer has access to a strategy that will keep his private consumption constant after small changes in government policy, *no matter what the other consumers do.* Where g_i^* is a consumer's contribution in the initial equilibrium, he can maintain the same consumption with tax rate τ_i and subsidy rate β, by setting $g_i = (g_i^* - \tau_i)/(1-\beta)$. As it happens, if all other consumers choose to neutralize the effects of the government policy on their own consumption, then it will be optimal for every consumer to do so.

In Game 1, if a consumer is to maintain his initial consumption as s_i and β change, then he must set his contributions at $g_i = (g_i^* - s_i\beta g_{\sim i})/(1 - \beta(1 - s_i))$, where g_i^* is i's no-tax equilibrium contribution. In contrast to Game 2, in order to neutralize the effects

[6] More generally, the same results would follow if the government committed to a modest positive contribution–so long as the amount it contributes is independent of any choices made by individuals.

of the tax-subsidy scheme on his own consumption, each consumer must know the total contributions $g_{\sim i}$ of others. Moreover, if each consumer believes the contributions of others to be invariant to his own contribution, then he will not choose the same net contribution as he would with no subsidy or taxes. It is possible to devise an extended version of Game 1 in such a way that an equilibrium exists in which a government tax-subsidy plan would be neutralized. But to do this, individuals have to be offered a richer strategy space than simply choosing an amount of contributions. The idea is to allow each individual to announce that his own contribution will be a "function" of the total contributions of others and to compute equilibrium as a fixed point of these functions. Specifically, suppose that every consumer $j \neq i$ announced that his own contributions would depend on the contributions of others according to the function $g_j = (g_j^* - s_j\beta g_{\sim j})/(1-\beta(1-s_j))$. Then every consumer other than i would maintain the same private consumption with the tax-subsidy scheme as without it. Furthermore, the best thing for consumer i to do would also be to maintain his original consumption, which he would accomplish by setting $g_i = (g_i^* - s_i\beta g_{\sim i})/(1 - \beta(1 - s_i))$.

So we see that even with the tax-subsidy policy outlined in Game 1, neutrality is possible if the game has an announcement stage and if individual strategies are expanded to allow individuals to make their contributions functions of the contributions of others. Does this restore the case for neutrality? We think not, for two reasons. One is a simple technical fact. In the game in which individual strategies are allowed to include functions, the neutral equilibrium is not the only Nash equilibrium. Another equilibrium for this same game is an equilibrium in which each individual submits a "constant" function, in which his contribution does not depend on the contributions of others, but only on the tax rate and the subsidy rate. The equilibrium that we originally calculated for Game 1 is also a Nash equilibrium for this game. These equilibria, as we have demonstrated in Theorem 2 are certain to be non-neutral with respect to the subsidy rate.

A second reason is that the government's intention in setting up a subsidy game is to influence the total amount of contributions. It would therefore try to set up the institutions of that game in such a way as to avoid an "announcement round" in which individuals could respond to each others' contributions. If public goods decisions are appropriately modeled as a one-shot game, for which the government can set up the rules, then there seems little doubt that it could set up the rules and information structure to approximate Game 1 and it would expect that subsidies unambiguously induce increased total contributions. But given that the game is played repeatedly, over the years, there may be room for lingering doubts that the insights gained from a one-shot model are the appropriate ones.

Repeated play of this game could be modeled as a "consistent conjectural variations equilibrium"[7] where consumers anticipate that eventually small changes in their contributions will be neutralized by offsetting actions of other consumers and accordingly choose to maintain constant consumptions in equilibrium, even with the tax scheme in Game 1. But in such a model, there will typically be many other subgame perfect Nash equilibria as

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[7] For discussions of consistent conjectural variations and related issues, see Laitner (1980), Bresnahan (1980), and Marschak and Selten (1978).

well, including the one-shot Nash equilibrium we found for Game 1.[8] As far as we know, no one has offered a model of repeated games in which neutralization of distortionary taxes stands out as a focal equilibrium among the many possible equilibria.

We have shown that if a government tax-subsidy scheme (whether lump-sum or not) collects more taxes from some individuals than they would have contributed voluntarily, then the scheme will change the real allocation of resources. Moreover, we demonstrated a simple tax-subsidy scheme that would necessarily increase the level of public goods even if nobody's taxes exceed her initial voluntary contributions. Although it is true that tax-subsidy schemes can be designed which will be neutralized if they collect no more from any individual than she would contribute voluntarily, it is also the case that a government that wants to increase the total amount of public goods can find a scheme to do so.

[8] MacMillan (1979) showed that in a repeated game of voluntary contributions, the set of subgame perfect equilibria is very large and contains Pareto optimal allocations. This plethora of equilibria is an instance of the well-known general problem in the theory of repeated games known as the "Folk Theorem."

Appendix

As in the text of the paper, let the demand functions $G_i(p,y)$ and $C_i(p,y)$ be the g and c respectively, that maximize $U_i(g,c_i)$ subject to $C_i + pG_i = y$. The first Lemma is useful in proving Theorems 1 and 3 on uniqueness of equilibrium.

Lemma A. *For either Game 1, or Game 2, suppose that for a given subsidy rate β and tax parameters, there are two Nash equilibria $g_1,\ldots,g_n$ and $g'_1,\ldots,g'_n$, such that $g' = \sum g'_i \geq g = \sum g_i$. Then for all i, $g'_{\sim i} \geq g_{\sim i}$.*

Proof of Lemma A. *We first prove the Lemma for Game 1. We show that $g'_{\sim i} \geq g_{\sim i}$ in each of the two possible cases where $g'_i > 0$ and $g_i = 0$.*

- Case (i). If $g'_i > 0$, then since $g'_1,\ldots,g'_n$ is a Nash equilibrium, it must be that $g' = G_i(p_i, m_i + (1-\beta)g'_{\sim i})$. Since $g_1,\ldots,g_n$ is a Nash equilibrium, it must be that $g \geq G_i(p_i, m_i + (1-\beta)g_{\sim i})$. By assumption, $g' \geq g$. Therefore $G_i(p_i, m_i + (1-\beta)g'_{\sim i}) \geq G_i(p_i, m_i + (1-\beta)g_{\sim i})$. Since the public good is assumed to be a normal good, it follows that $m_i + (1-\beta)g'_{\sim i} \geq m_i + (1-\beta)g_{\sim i}$ and hence $g'_{\sim i} \geq g_{\sim i}$.
- Case (ii). Consumer i has $g'_i = 0$. Then $g_{\sim i} \leq g \leq g' = g'_{\sim i}$ so that $g'_{\sim i} \geq g_{\sim i}$.

The proof of Lemma A for Game 2 uses the same argument except that this time we replace m_i by $m_i - \tau_i$ at each point in the proof. ∎

Proof of Theorem 1.

The existence of at least one Nash equilibrium is a straightforward application of Brouwer's fixed point theorem to the function mapping a vector is of contributions $g_1,\ldots,g_n$ into the vector of "best responses." See, for example, Bergstrom, Blume, and Varian (1986).

Now let us consider uniqueness. Suppose that there are two distinct Nash equilibria corresponding to a given β. Let $g_1,\ldots,g_n$, $c_1,\ldots,c_n$, and $g'_1,\ldots,g'_n$, $c'_1,\ldots,c'_n$ be the individual contributions and private consumptions in the two equilibria. Suppose, without loss of generality that $g' = \sum g'_i \geq g = \sum g_i$. Then according to Lemma A, it must be that for all i, $g'_{\sim i} \geq g_{\sim i}$ and since the two equilibria are assumed to be *distinct*, it must be that $g'_{\sim i} > g_{\sim i}$ for some i and therefore $g' > g$. [9]

Let $p_i = 1 - \beta(1 - s_i)$. Since the private good is a normal good and $g'_{\sim i} \geq g_{\sim i}$, it must be that $c'_i = C_i(p_i, m_i + (1-\beta)g'_{\sim i}) \geq C_i(p_i, m_i + (1-\beta)g_{\sim i}) = c_i$ for all i with strict inequality for some i. Therefore, $\sum_i c'_i > \sum_i c_i$. But in equilibrium, it must be that $\sum_i c'_i + g' = \sum_i m_i = \sum_i c_i + g$.[10] Since $\sum_i c'_i > \sum_i c_i$, it must be that $g' < g$. But this contradicts our assumption that $g' \geq g$. It follows that there can not be two distinct Nash equilibria. ∎

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[9] The result that $g' > g$ follows from $g = (\sum g_{\sim i})/(n-1) > (\sum g'_{\sim i})/(n-1) = g'$.

[10] This is just the feasibility constraint. To see that it is always holds in Nash equilibrium, one adds the budget equations over all individuals and makes appropriate simplifications.

Proof of Theorem 3.

The proof Theorem 3 is very similar to the proof of Theorem 1. The first two paragraphs of that proof carries over unchanged. The third paragraph also applies with the following modifications. For all i, set $p_i = (1-\beta)$ and wherever m_i appears in the argument, replace it by $m_i - \tau_i$. ∎

Proof of Theorem 5.

Let c_i^*, ℓ_i^* and g_i^*, be the choices made by consumer i in the no-tax equilibrium. In Stage 3, consumer i chooses c_i^* and g_i^* to maximize $U_i(c_i, g_{\sim i}^* + g_i, \ell_i^*)$ subject to the constraint $c_i + g_i = m_i(\ell_i^*)$ and subject to $g_i^* \geq 0$. This problem can be expressed equivalently as choosing g and c_i so as to maximize $U_i(c_i, g, \ell_i)$ subject to $c_i + g = m_i(\ell_i^*) + g_{\sim i}^*$ and subject to $g \geq g_{\sim i}^*$.

Now let the government collects taxes according to the schedule $t_i(\ell)$, from each i and spend its revenue on the public good. We claim that there is an equilibrium where each consumer reduces his voluntary contributions to exactly offset the new taxes. That is, each consumer reduces his contribution from g_i^* to $g_i^* - t_i(\ell^*)$, which is non-negative by assumption. If in Stage 2, consumers choose ℓ^*, then in Stage 3, consumer i's budget constraint will be $c_i + g_i = m_i(\ell_i^*) - t_i(\ell^*)$ and $g_i \geq 0$. The total amount of public goods is the sum paid for out of taxes and individual contributions. Therefore $g = g_i + t_i(\ell) + \sum_{j \neq i}(g_j + t_j(\ell))$.

If all consumers other than i contribute $g_j = g_j^* - t_j(\ell^*)$, then this last equation is equivalent to $g = g_i + t_i + g_{\sim i}^*$. Therefore the budget constraint becomes $c_i + g = m_i(\ell^*) + g_{\sim i}^*$ and the constraint that $g_i \geq 0$ is equivalent to $g \geq g^* \sim i + t_i(\ell^*)$. But this budget is the same as the budget constraint in the absence of the tax. Consumer i can restore the no-tax equilibrium by choosing $c_i = c_i^*$ and $g_i = g_i^* - t_i(\ell^*)$, in which case the total supply of public goods is $g^* = \sum_1^n g_i^*$, just as in the no-tax equilibrium. Not only is this a possible choice for consumer i, it is the best affordable choice for consumer i given the actions of the other consumers. This follows from the "principle of revealed preference" and from the fact that the budget set available to consumer i with the taxes is contained the budget set available in the no-tax equilibrium.

This reasoning shows that for any vector ℓ chosen in Stage 2, the same equilibrium will obtain with the tax as without the tax. Therefore the choices of labor supply in Stage 2 will not be altered by the tax. Given that the labor supply in Stage 2 does change, we have just shown that the choices in Stage 3 exactly offset the tax and hence we have the promised neutrality. ∎

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